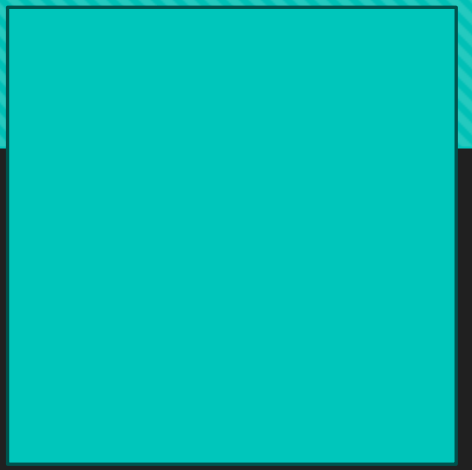
Polynomials

Class 10



What is a Polynomial?

$\frac{1}{x} \Rightarrow (1)x^{-1}$ → Not acceptable

x

$ax^n + bx^{n-1} + cx^{n-2} + \cdots$

$a, b, c \cdots \rightarrow$ coefficients (Real no.)

$x \rightarrow$ variable

$n \rightarrow$ exponent

Natural no

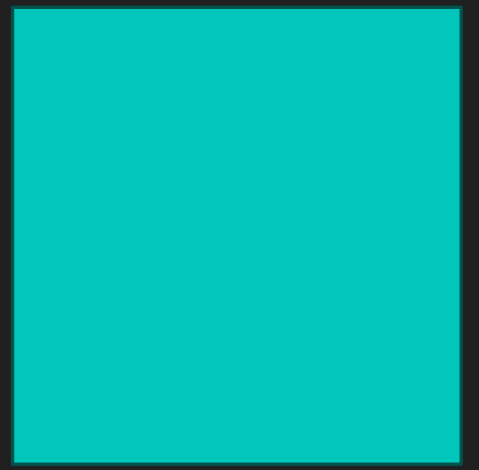
What is a Polynomial?

$x^4 - 10x^2 - \sqrt{10}x + 11$

4, 2, 1

$\frac{1}{x-3}$, $\sqrt{x} - 4x^2$

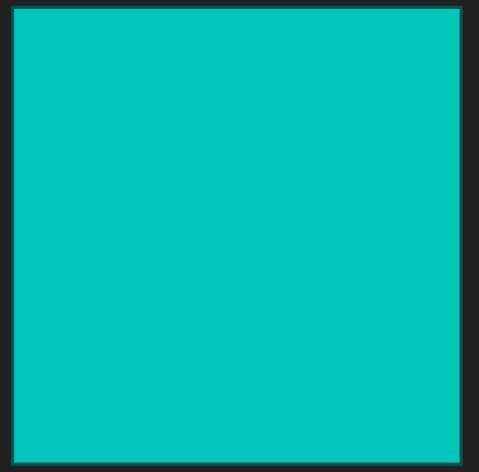
$(x)^{1/2}$ x



Degree of a Polynomial

Highest exponent of the variable

$$ax^n + bx^{n-1} + cx^{n-2} + \dots$$



Degree of a Polynomial

$x^5 - x^6 + \frac{1}{2} + 4x$

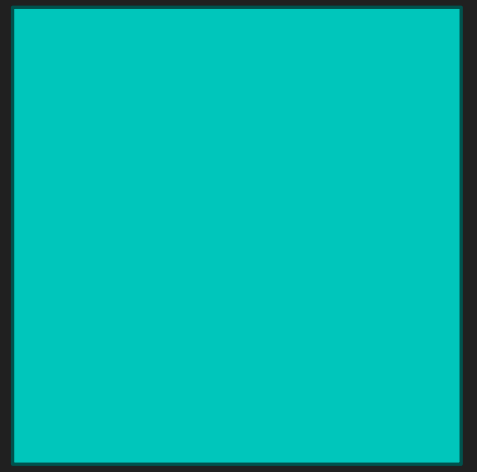
5 6 1

Linear Polynomial → Degree of poly = 1

Degree = 1

$x + 4$

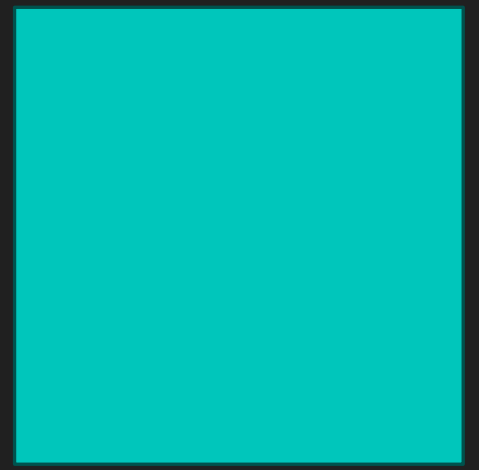
1



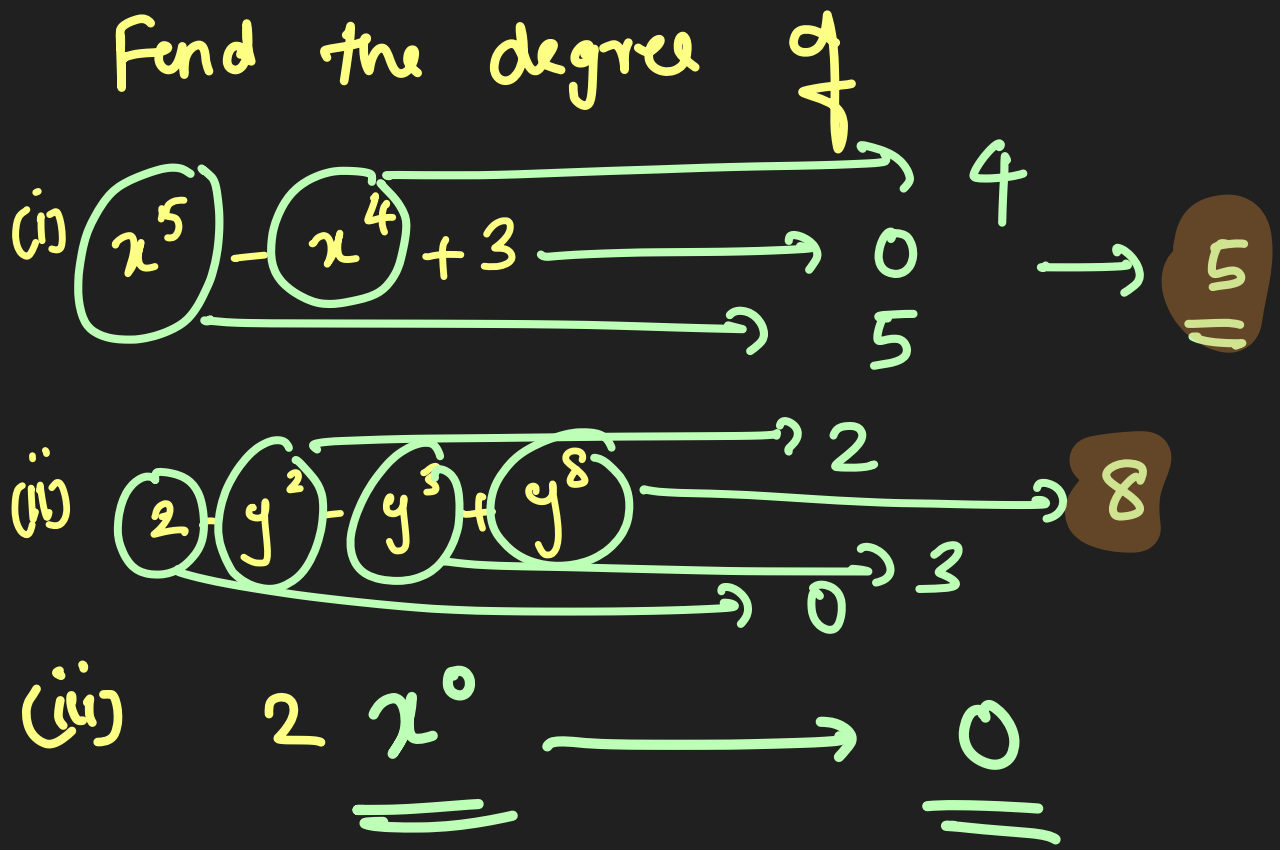
Quadratic/Cubic Polynomial

degree = 2

degree = 3



Additional Problems



Additional Problems

State whether foll. exp are polynomials or not?

(i) $4x^2 - 3x + 7 \longrightarrow$ ✓

(ii) $y^2 + \sqrt{2} \longrightarrow$ ✓

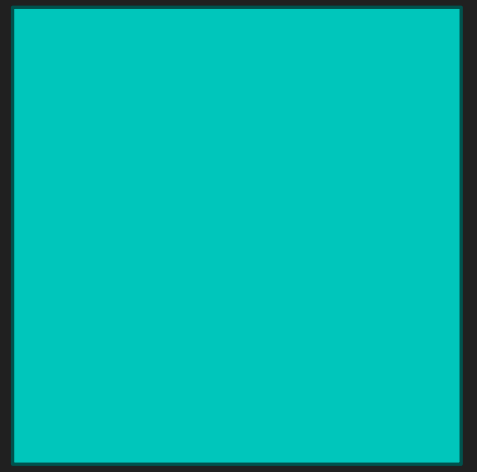
(iii) $y + 2/y \longrightarrow$ ✗

(iv) $3\sqrt{t} + \sqrt{2}\,t \longrightarrow$ ✗

Coef ☐

variable ☐

natural ☐



Additional Problems

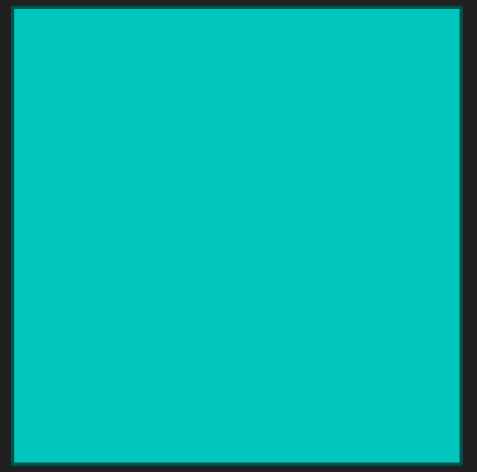
Degree of a Polynomial

(i) $x^2 + x \longrightarrow 2$

(ii) $x - x^3 \longrightarrow 3$

(iii) $7x^3 \longrightarrow 3$

(iv) $y + y^2 + 4 \longrightarrow 2$



Additional Problems

$4x + 3$

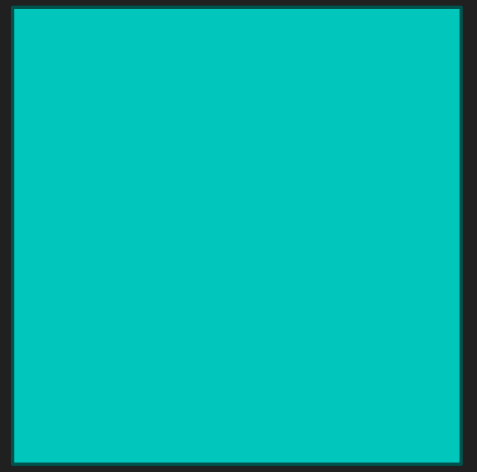
Identify linear, cubic, quad

(i) $x^2 + x \longrightarrow$ Quad

(ii) $x - x^3 \longrightarrow$ Cubic

(iii) $7x^3 \longrightarrow$ Cubic

(iv) $y + y^2 + 4 \longrightarrow$ Quad



Value of a Polynomial →

value of $p(x)$ for a given value of x.

$p(x)$ for a given value of x.

$p(x) = x^3 - 4x^2 + 10x$

$x = 2$ Find the value of $p(x)$

$p(2) = 2^3 - 4 \times 2^2 + 10 \times 2 = 8 - 16 + 20$

$= 12$

Zero of a Polynomial

at $x = 5$, $p(x) = 0$

$x = 5$ ← at $x = 5$, $2x - 10 = 0$

value of the variable (value of x)

for which $p(x) = 0$

$p(x) : 2x - 10$

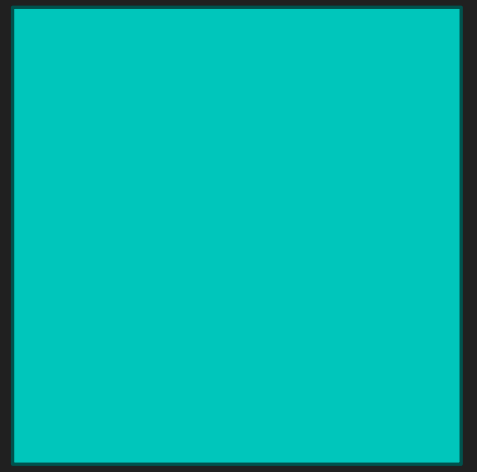
$p(x) = 0$

$2x - 10 = 0$

$2x = 10$

$x = \frac{10}{2}$

$= 5$

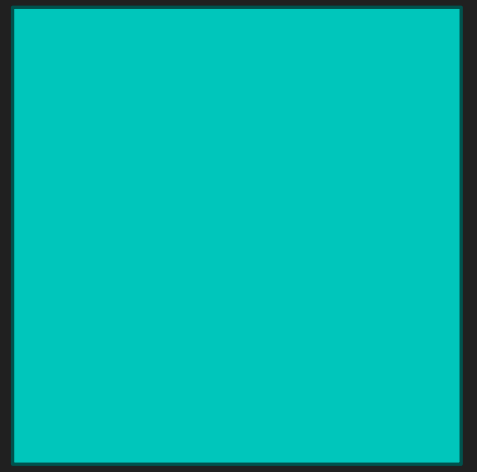


Zero of Polynomial – Contd.

$x = 5$ is the zero of the polynomial $2x - 10$.

$p(x) = x^2 - 5x + 6$

$p(x) = 0$



$x = 2$ and $x = 3$ are the zeros of the polynomial $x^2 - 5x + 6$

Zero of Polynomial – Contd.

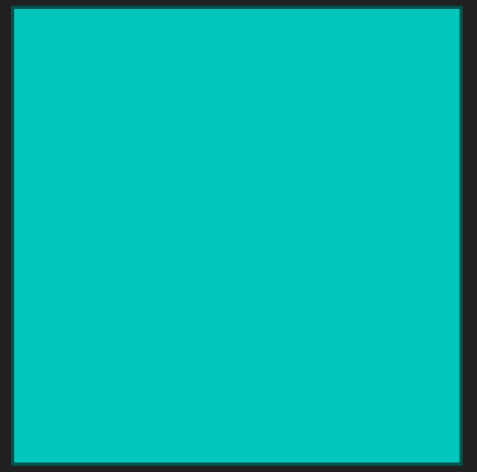
$$x^2 - 5x + 6 = 0$$

$$x^2 - 3x - 2x + 6 = 0$$

$$x(x-3) - 2(x-3) = 0$$

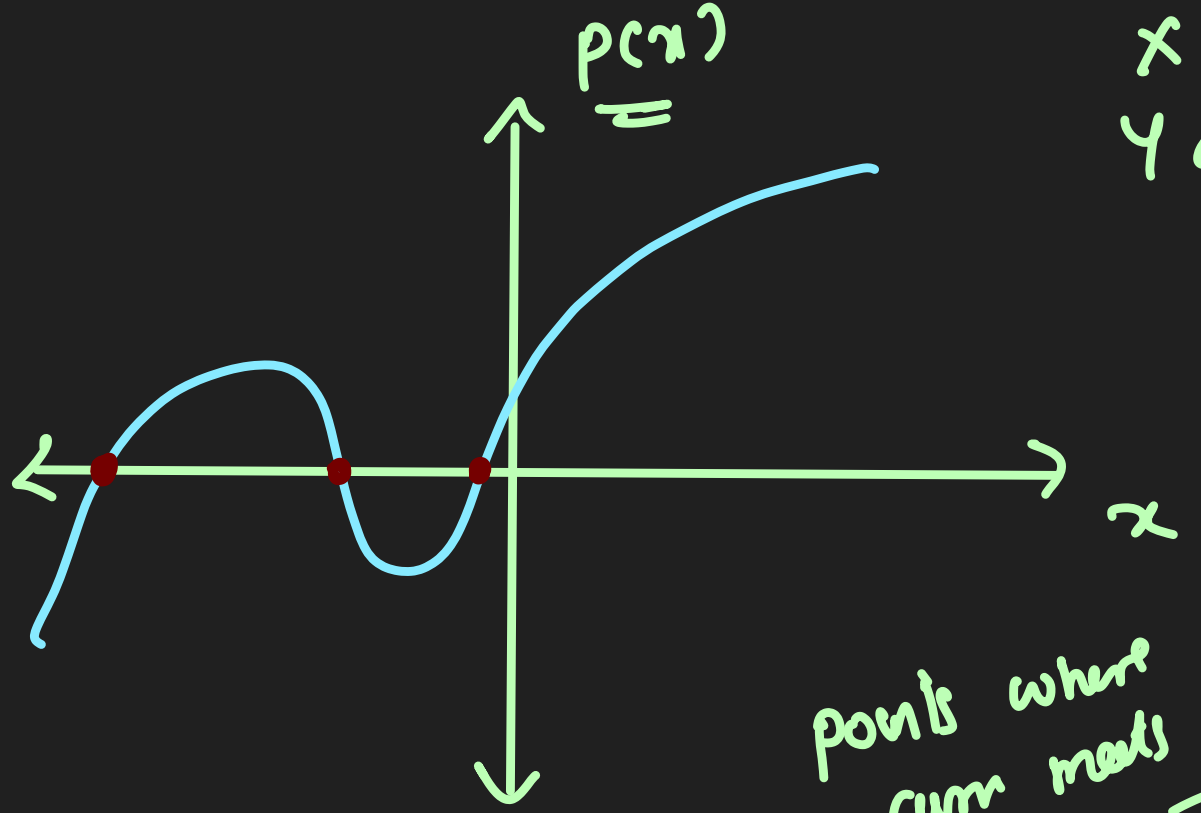
$$(x-3)[x-2] = 0$$

$$x = 2, 3$$



graphical method

Geometric Meaning – Zero of Polynomial

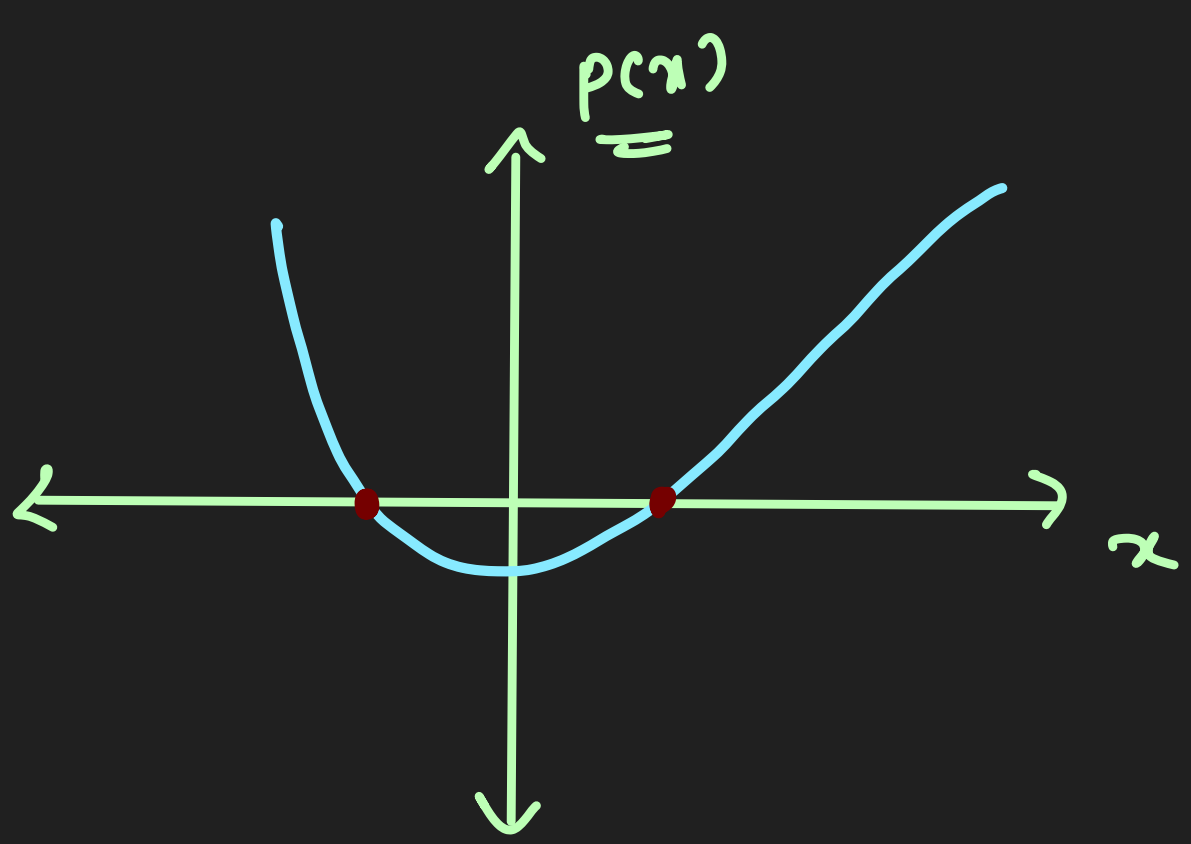


x axis → variable

y axis → value of function

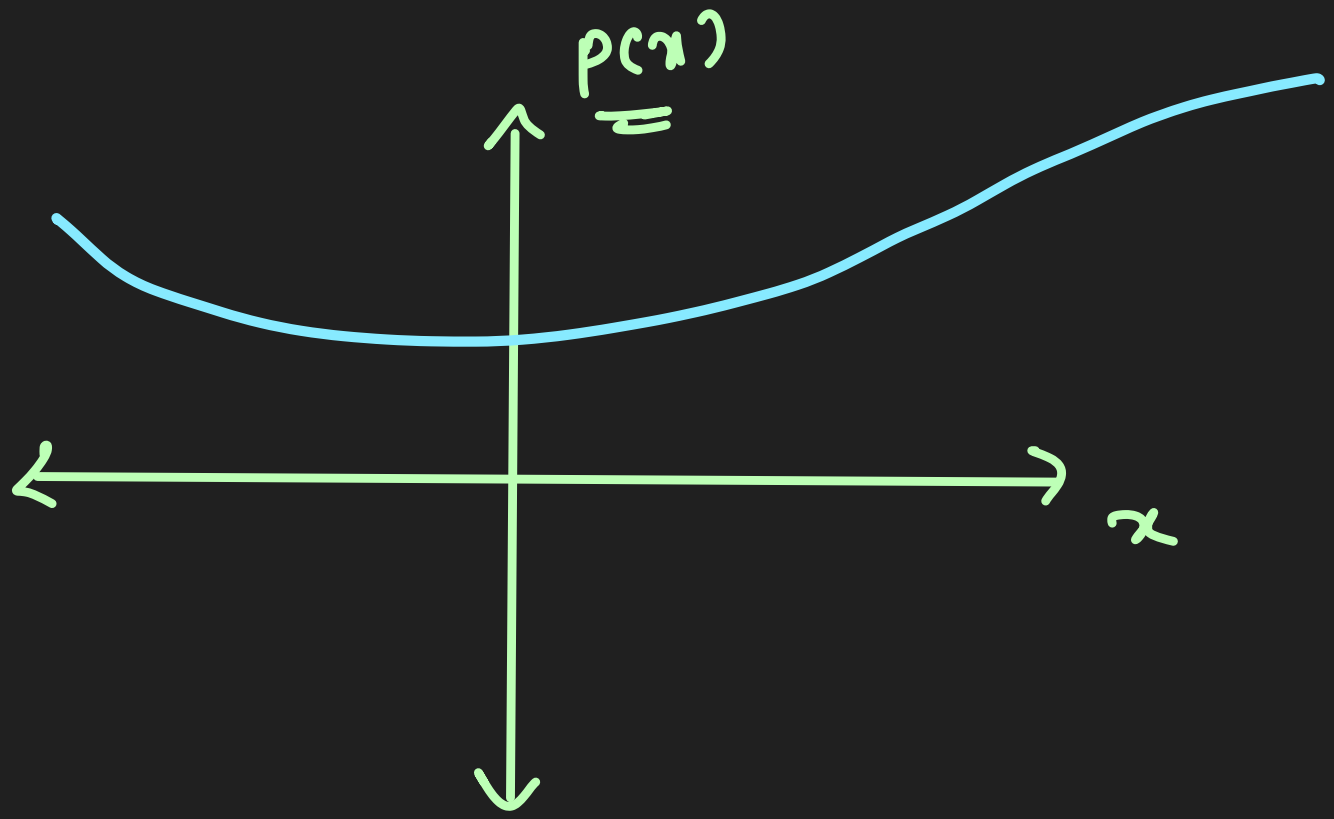
points where curve meets x axis

Geometric Meaning – Contd.



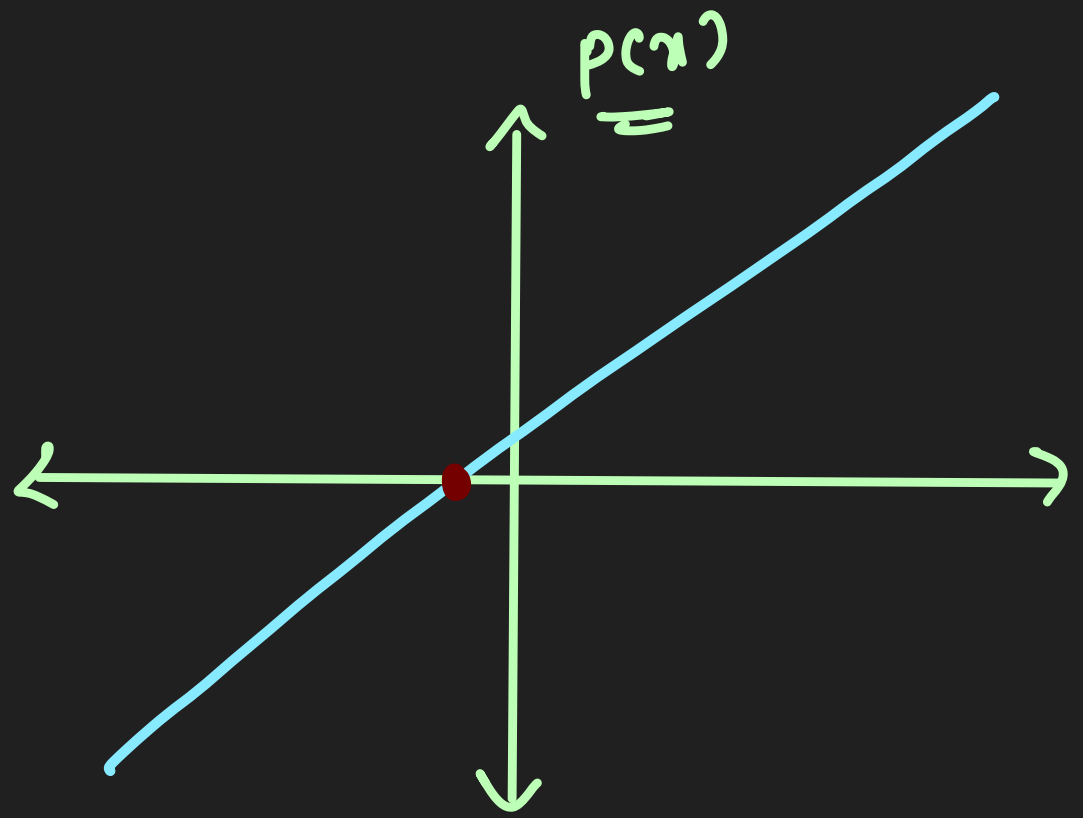
2 zeros for this polynomial

Geometric Meaning – Contd.



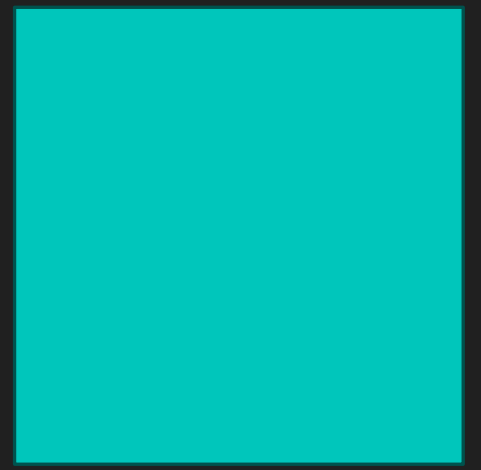
The curve is not meeting X axis

Geometric Meaning – Contd.



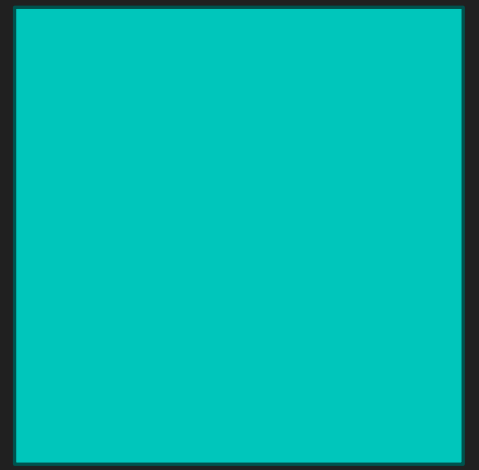
Here
a polynomial

Geometric Meaning – Contd.



Sum / Product of Zeros

Relationship betn
Zeros of polynomial
and its coefficients



Sum / Product of Zeros

$ax^2 + bx + c = 0$ → Quadratic poly

$ax + b = 0$ → linear

↓

zero of poly : $-b/a$

Sum / Product of Zeros

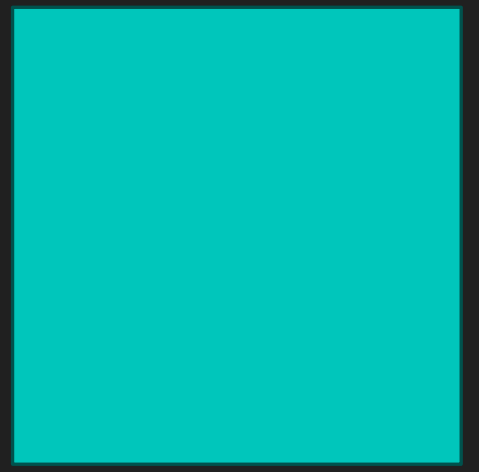
$\alpha : \alpha, \beta$.

$ax^2 + bx + c = 0$

α, β are zeros of the polynomial

$\alpha + \beta = -b/a$

$\alpha\beta = c/a$



Exercise 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$ (ii) $4s^2 - 4s + 1$ (iii) $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$ (v) $t^2 - 15$ (vi) $3x^2 - x - 4$

Exercise 2.2

$\alpha = 4, \quad \beta = -2$

$\alpha + \beta = -b/a \qquad \alpha\beta = c/a$

$x^2 - 2x - 8 = 0$

$-8, \quad -2$

$-4, 2$

$x^2 - 4x + 2x - 8 = 0$

$x(x-4) + 2(x-4) = 0$

$(x-4)(x+2) = 0$

$x = 4, -2$

$4 + (-2) = 2 = -\frac{(-2)}{1} = \frac{2}{1}$

$4x - 2 = -8$

$= -\frac{8}{1} = -8$

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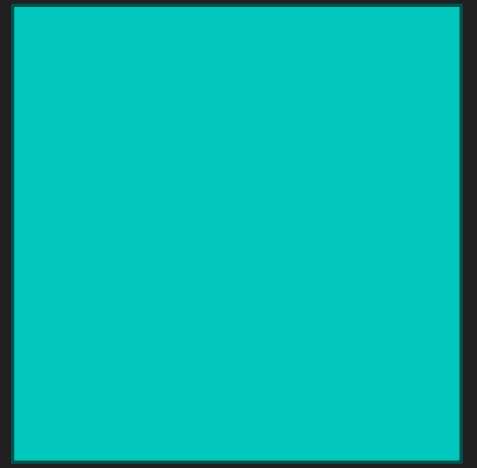

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(vi) $3x^2 - x - 4$

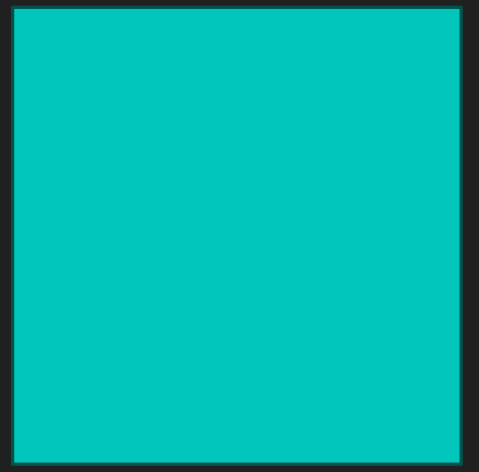
Exercise 2.2

$$ax^3 + bx^2 + cx + d = 0$$

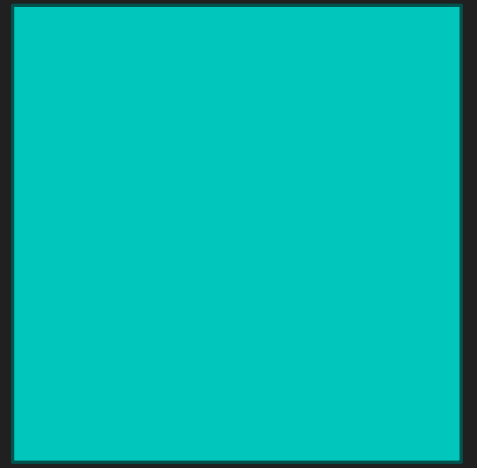
$$\alpha + \beta + \gamma = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$$

$$\alpha\beta\gamma = -d/a$$



Division of a Polynomial



Division of a Polynomial