

Polynomials

Class 10



What is a Polynomial?

$$\frac{1}{x} \Rightarrow (1)x^{-1}$$

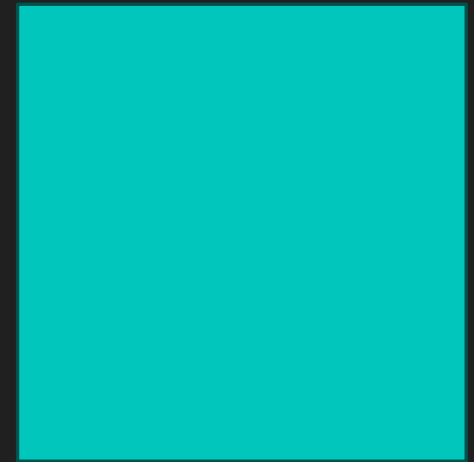
Not acceptable

$$ax^n + bx^{n-1} + cx^{n-2} + \dots$$

$a, b, c, \dots \rightarrow$ coefficients
(Real no.)

$x \rightarrow$ variable

$n \rightarrow$ exponent
Natural no.



What is a Polynomial?

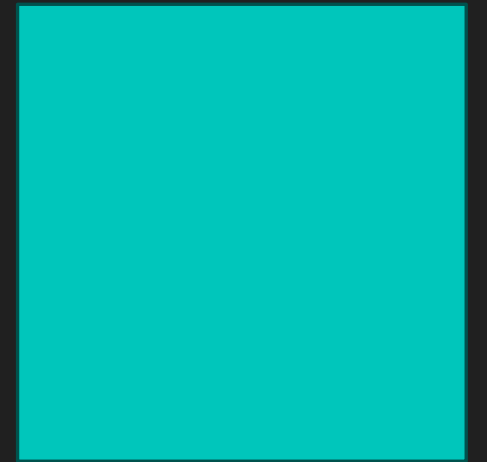
$$x^4 - 10x^2 - \sqrt{10}x + 11$$

Handwritten annotations above the equation: arrows pointing from the exponents 4, 2, and 1 to their respective terms.

$$\frac{1}{x-3}, \quad \sqrt{x} - 4x^2$$

$$\underline{\underline{(x)^{-1/2}}}$$

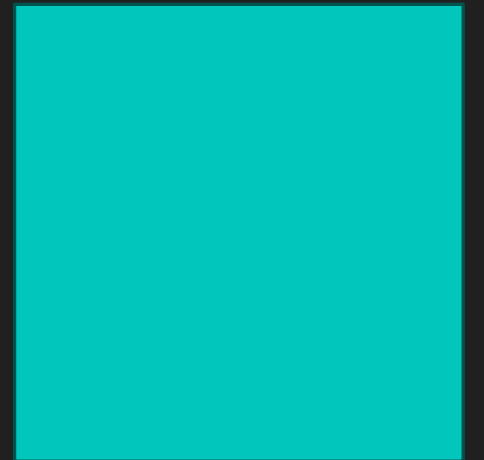
Handwritten annotations: a double underline under the expression and a checkmark to its right.



Degree of a Polynomial

highest exponent of the
variable

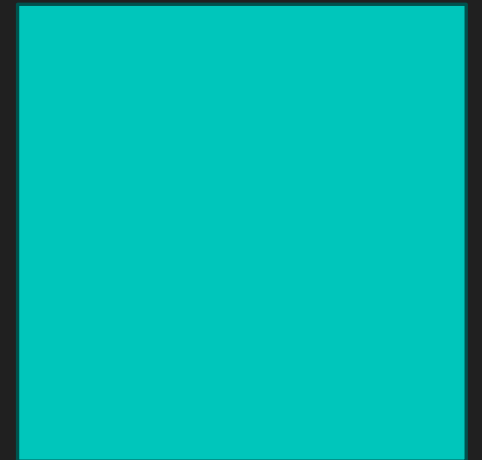
$$ax^n + bx^{n-1} + cx^{n-2} + \dots$$



Degree of a Polynomial

$$x^5 - x^6 + \frac{1}{2} + 4x$$

Diagram illustrating the degree of a polynomial. The polynomial is $x^5 - x^6 + \frac{1}{2} + 4x$. The degrees of the terms are indicated by arrows pointing to the exponents: 5 for x^5 , 6 for x^6 , and 1 for $4x$. A horizontal line is drawn below the exponents, indicating that the degree of the polynomial is the highest of these values, which is 6.



Linear Polynomial



Degree of poly = 1

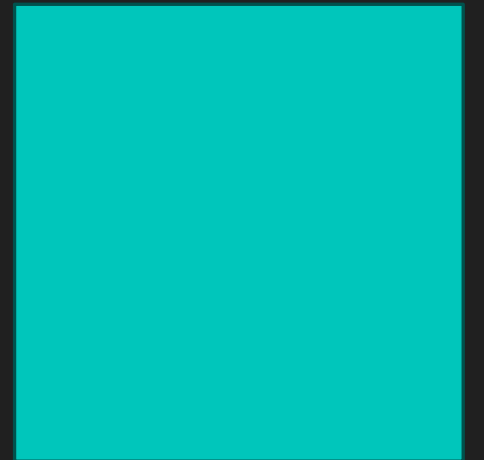
Degree = 1

$x + 4$

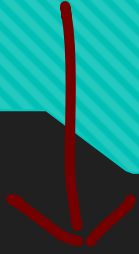
+

\downarrow

1



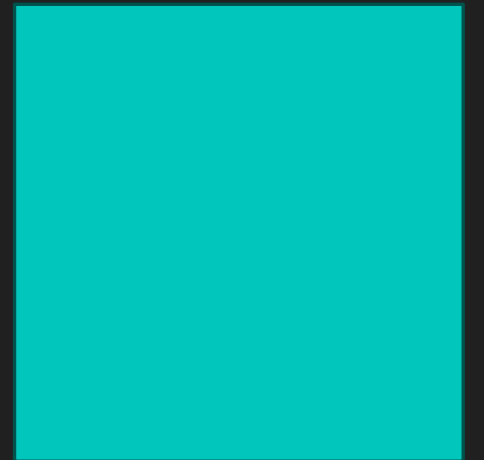
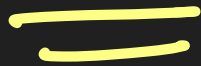
Quadratic/Cubic Polynomial



degree = 2



degree = 3



Additional Problems

Find the degree of

(i) $x^5 - x^4 + 3$

Diagram showing the degree of each term: x^5 has degree 5, x^4 has degree 4, and the constant term 3 has degree 0. The highest degree is 5, which is the degree of the polynomial.

5

(ii) $2y^2 - y^3 + y^8$

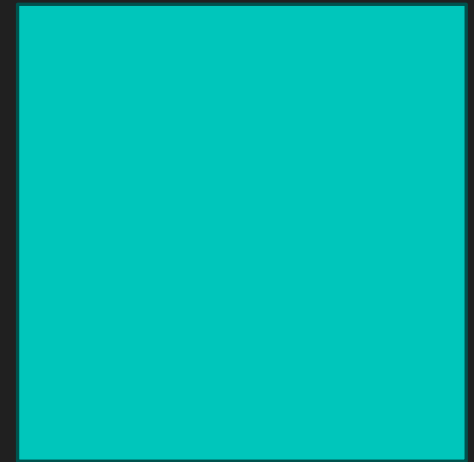
Diagram showing the degree of each term: $2y^2$ has degree 2, y^3 has degree 3, and y^8 has degree 8. The highest degree is 8, which is the degree of the polynomial.

8

(iii) $2x^0$

Diagram showing the degree of the term: $2x^0$ has degree 0. The degree of the polynomial is 0.

0



Additional Problems

State whether foll. exp are
polynomials or not?

(i) $4x^2 - 3x + 7 \rightarrow \checkmark$

(ii) $y^2 + \sqrt{2} \rightarrow \checkmark$

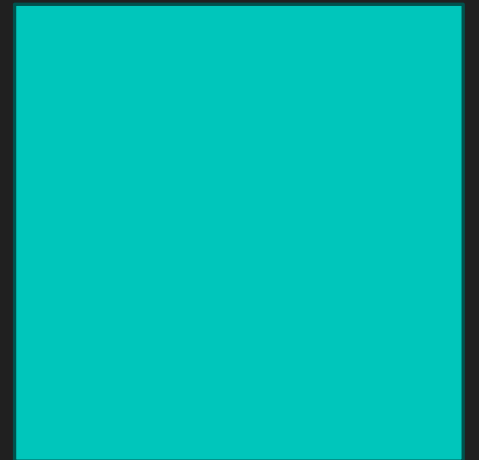
(iii) $y + 2/y \rightarrow \times$

(iv) $3\sqrt{t} + \sqrt{2}t \rightarrow \times$

coef

variable

natural



Additional Problems

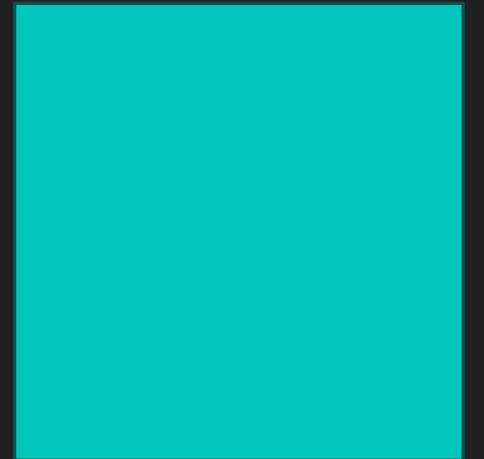
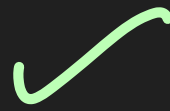
Degree of a Polynomial

$$(i) \quad x^2 + x \longrightarrow 2$$

$$(ii) \quad x - x^3 \longrightarrow 3$$

$$(iii) \quad 7x^3 \longrightarrow 3$$

$$(iv) \quad y + y^2 + 4 \longrightarrow \underline{\underline{2}}$$



Additional Problems

$$\underline{\underline{4x + 3}}$$

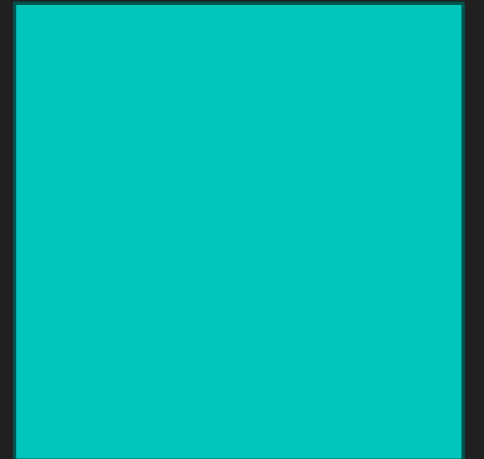
Identify linear, cubic, quad

(i) $x^2 + x \rightarrow$ Quad

(ii) $x - x^3 \rightarrow$ Cubic

(iii) $7x^3 \rightarrow$ Cubic

(iv) $y + y^2 + 4 \rightarrow$ Quad



Value of a Polynomial →

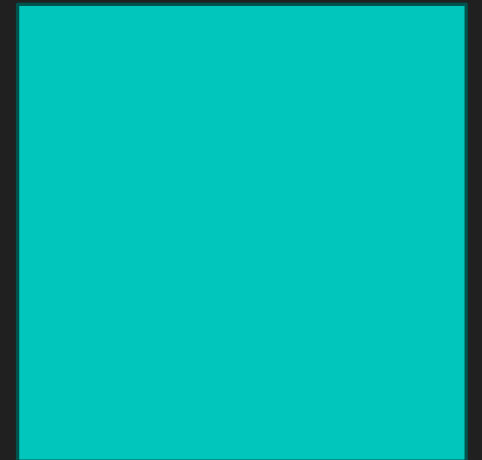
value of $p(x)$ for
a given value of x .

$p(x)$ for a given value of x .

$$p(x) = x^3 - 4x^2 + 10x$$

$x = 2$ Find the value of $p(x)$

$$\begin{aligned} p(2) &= 2^3 - 4 \times 2^2 + 10 \times 2 = 8 - 16 + 20 \\ &= \underline{\underline{12}} \end{aligned}$$



Zero of a Polynomial

$$x = 5 \uparrow$$

$$\text{at } x = 5, p(x) = 0$$

$$\leftarrow \text{at } x = 5, \quad 2x - 10 = 0$$

value of the variable (value of x)
for which $p(x) = 0$

$$p(x) = 2x - 10$$

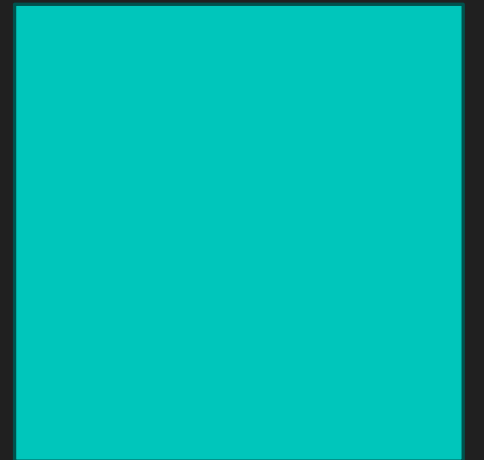
$$\underline{\underline{p(x) = 0}}$$

$$2x - 10 = 0$$

$$2x = 10$$

$$x = \frac{10}{2}$$

$$= \underline{\underline{5}}$$

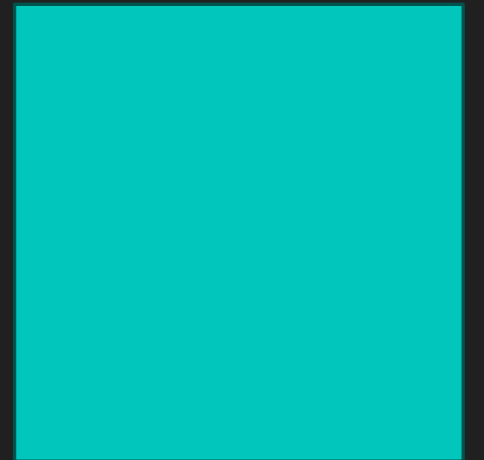


Zero of Polynomial – Contd.

$x=5$ is the zero of the
polynomial $2x-10$.

$$p(x) = x^2 - 5x + 6$$

$$p(x) = 0$$



$x=2$ and $x=3$ are the zeros of the polynomial

Zero of Polynomial – Contd.

$$\underline{\underline{x^2 - 5x + 6}}$$

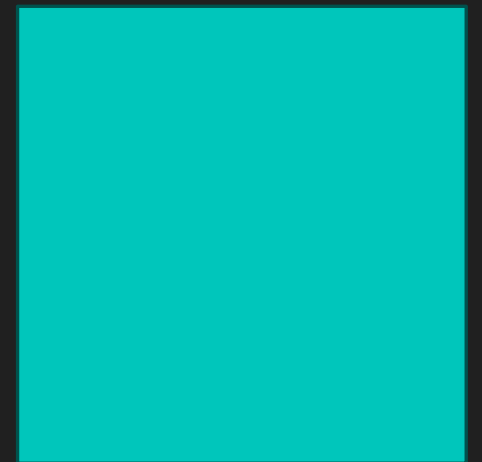
$$x^2 - 5x + 6 = 0$$

$$x^2 - 3x - 2x + 6 = 0$$

$$x(x-3) - 2(x-3) = 0$$

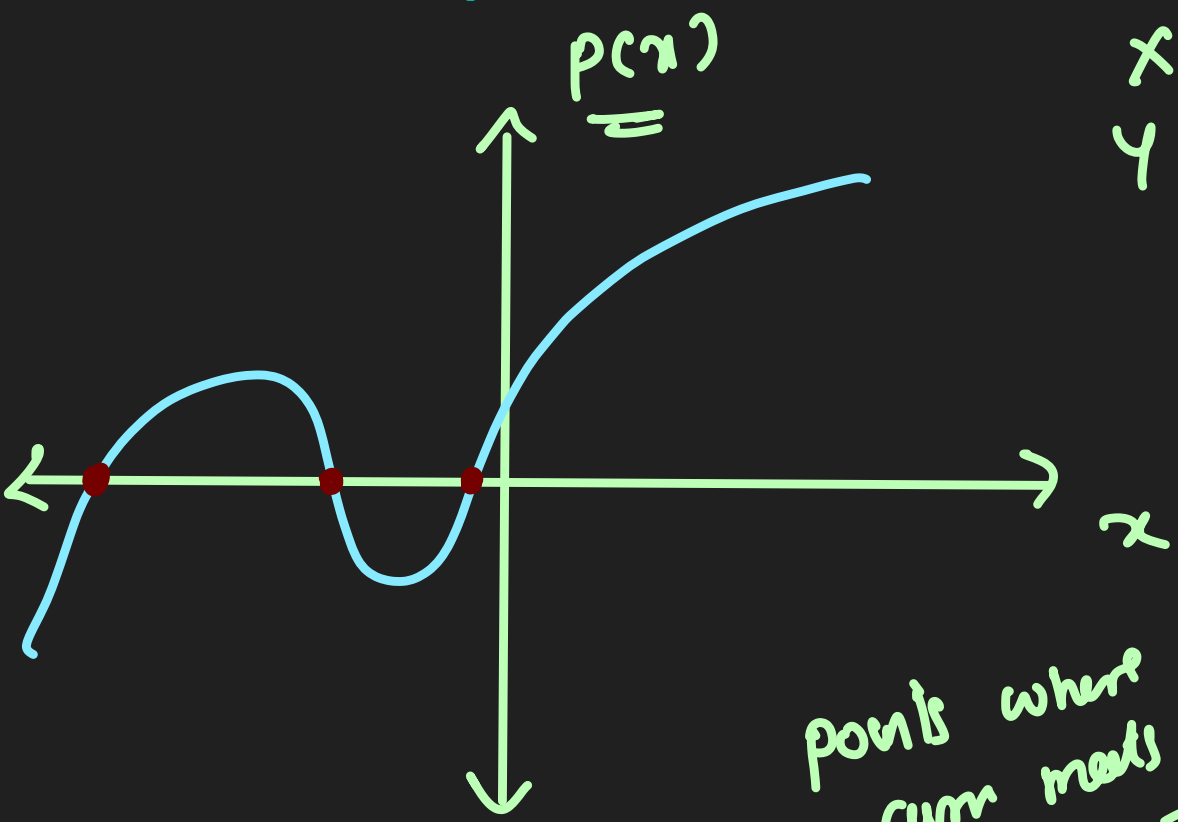
$$(x-3)[x-2] = 0$$

$$x = \underline{\underline{2, 3}}$$



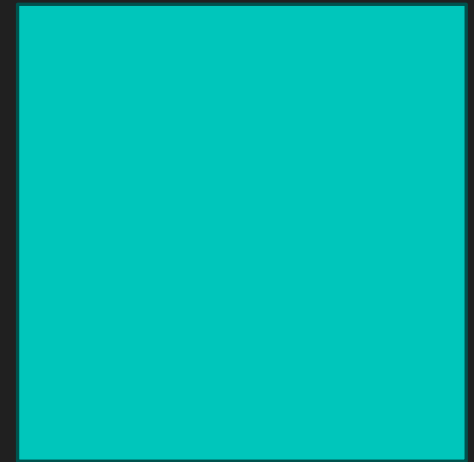
graphical method

Geometric Meaning – Zero of Polynomial



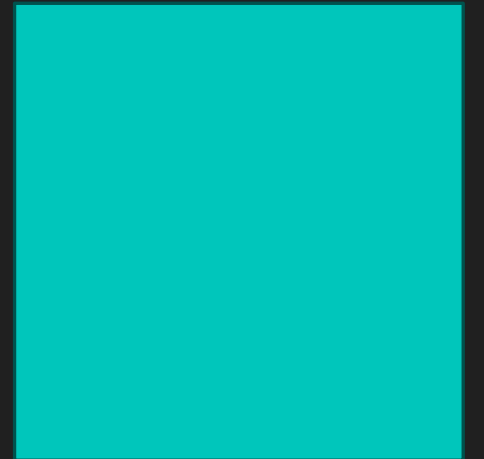
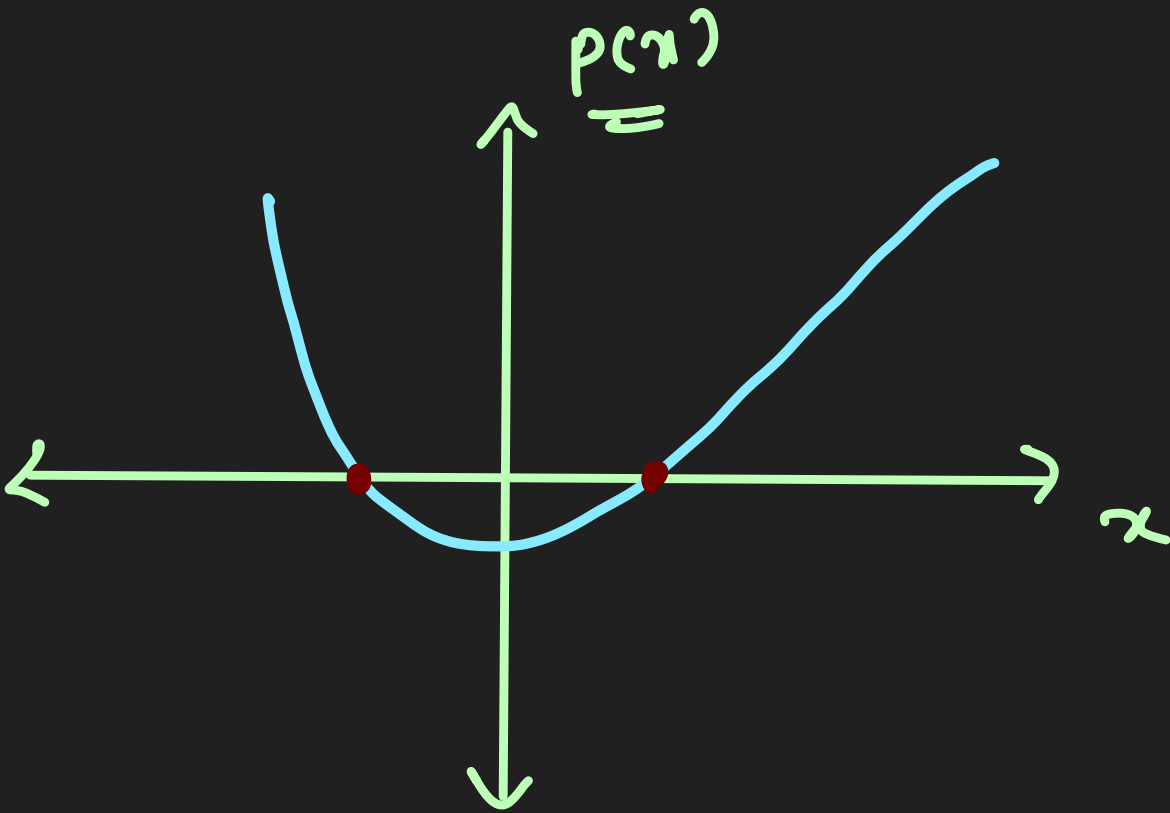
x axis → variable
y axis → value of function

points where
curve meets x axis

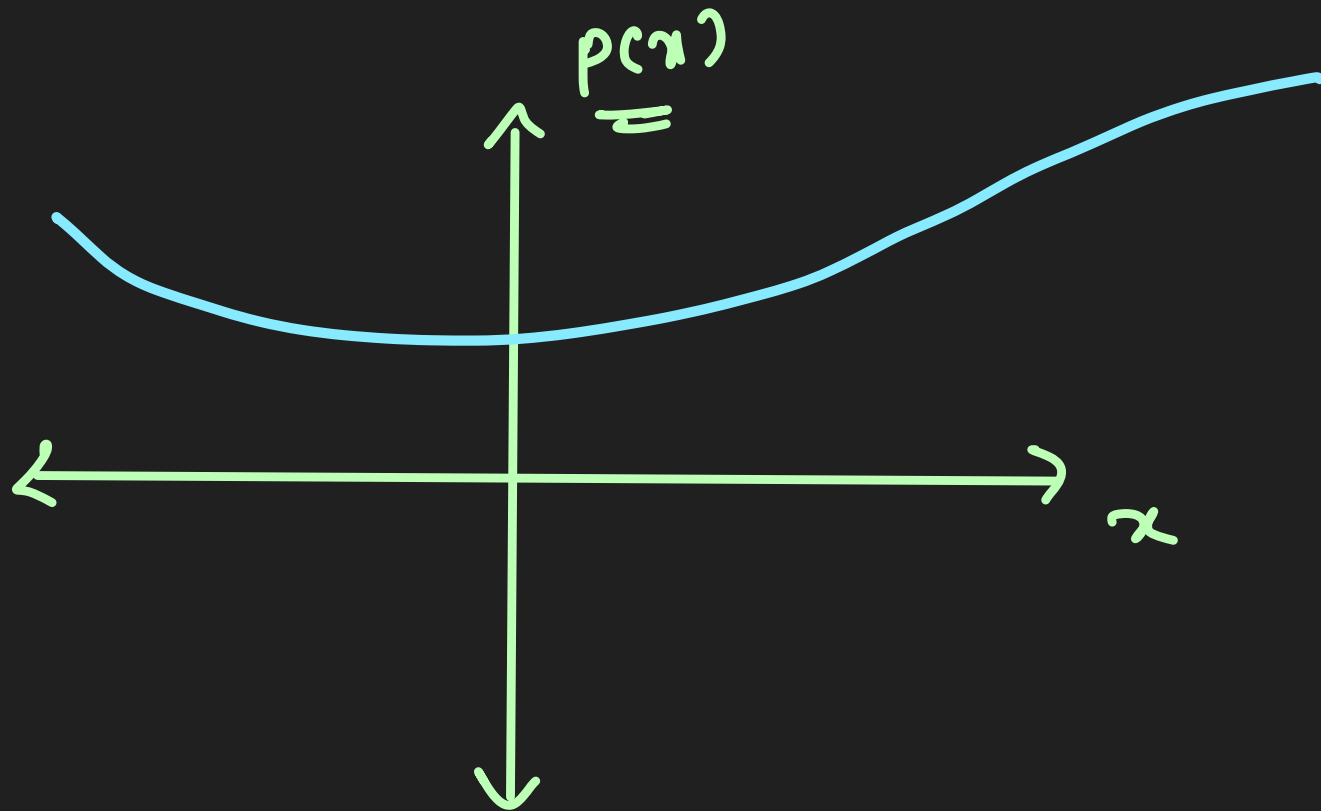


Geometric Meaning – Contd.

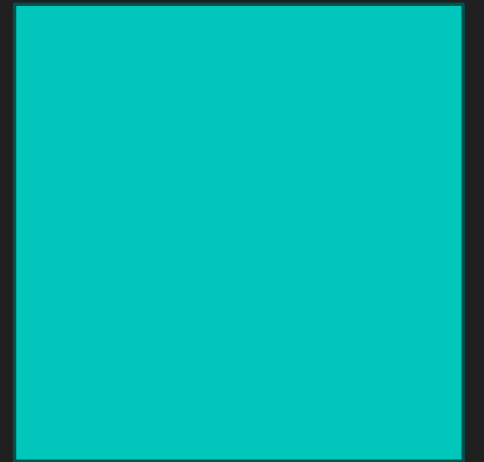
2 zeros for this
polynomial



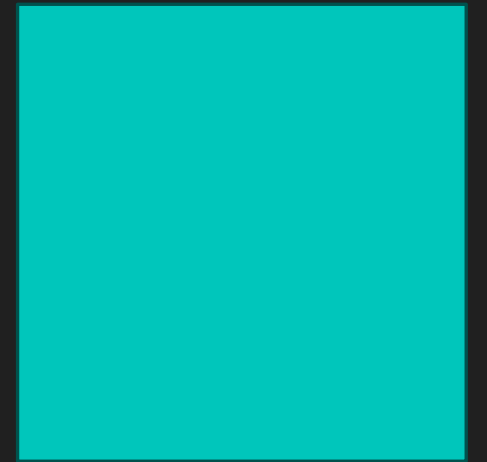
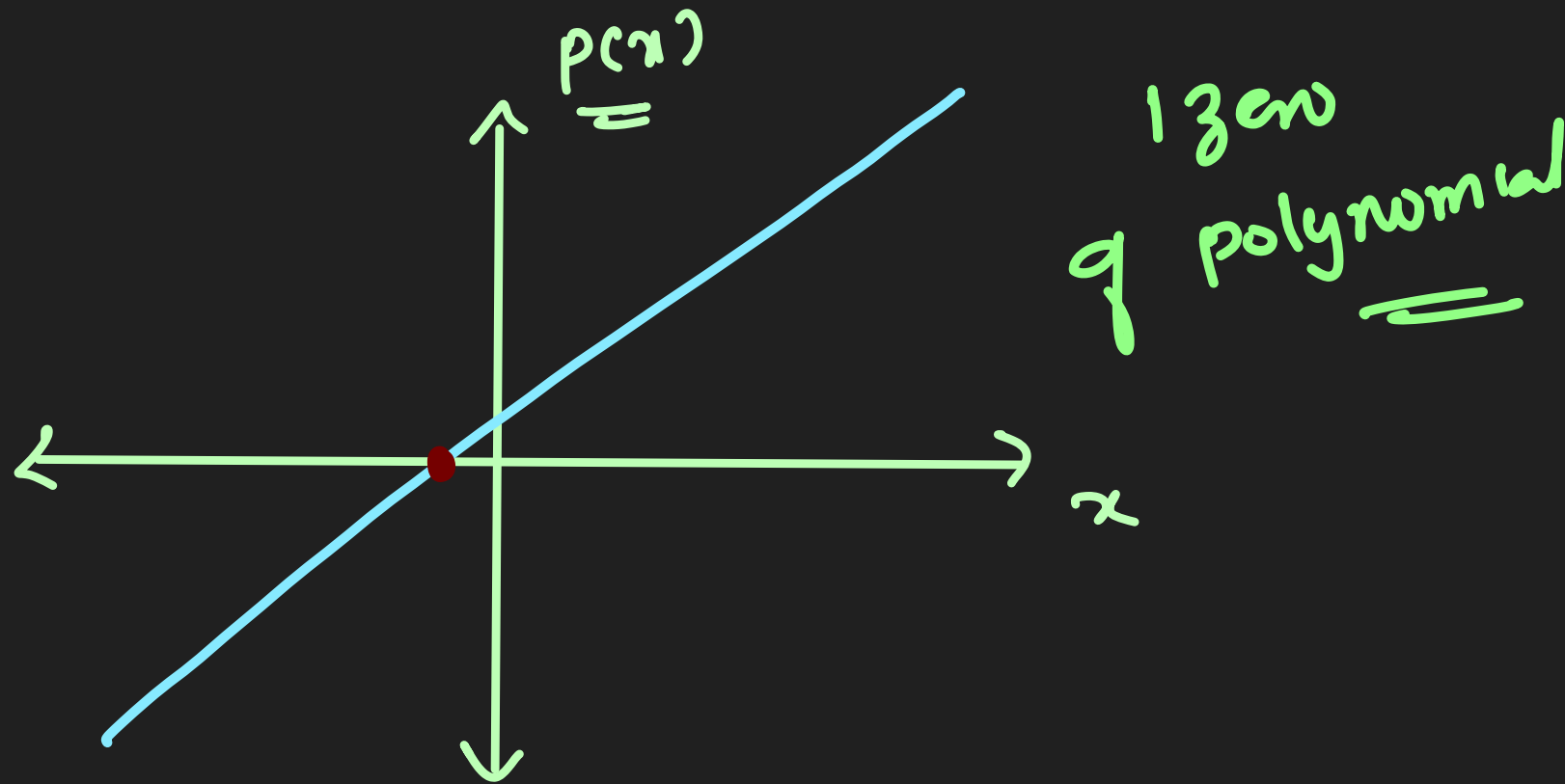
Geometric Meaning – Contd.



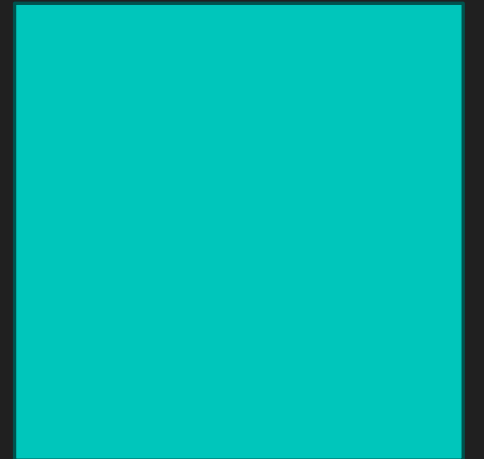
The curve is not
meeting x axis



Geometric Meaning – Contd.

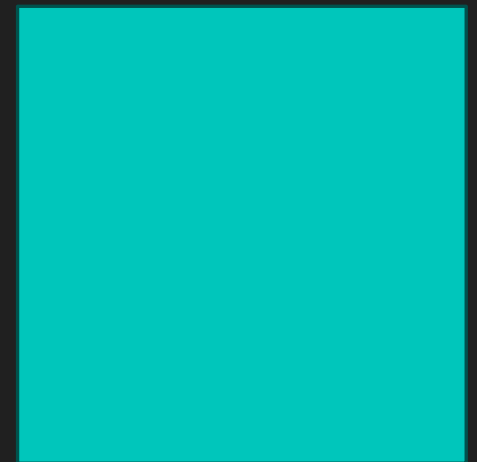


Geometric Meaning – Contd.



Sum / Product of Zeros

Relationship bet'n
zeros of polynomial
and its coefficients



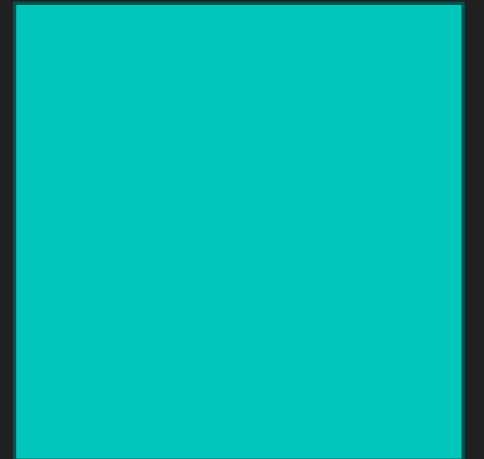
Sum / Product of Zeros

$$ax^2 + bx + c = 0 \rightarrow \text{Quadratic poly}$$

$$ax + b = 0 \rightarrow \text{Linear}$$



$$\text{zero of poly} = \underline{\underline{-b/a}}$$



Sum / Product of Zeros

$$\alpha = \alpha, \beta.$$

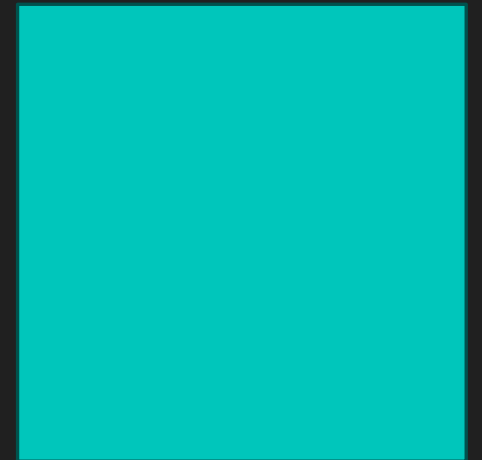
$$ax^2 + bx + c = 0$$

$$\alpha + \beta = -b/a$$

$$\alpha\beta = c/a$$

α, β are zeros of
the polynomial

==



Exercise 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

(ii) $4s^2 - 4s + 1$

(iii) $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$

(v) $t^2 - 15$

(vi) $3x^2 - x - 4$

Exercise 2.2

$$\alpha = 4, \quad \beta = -2$$

$$\alpha + \beta = -b/a \quad \alpha\beta = \frac{c}{a}$$

$$x^2 - 2x - 8 = 0$$

$$-8, -2$$

✓

$$x^2 - 4x + 2x - 8 = 0$$

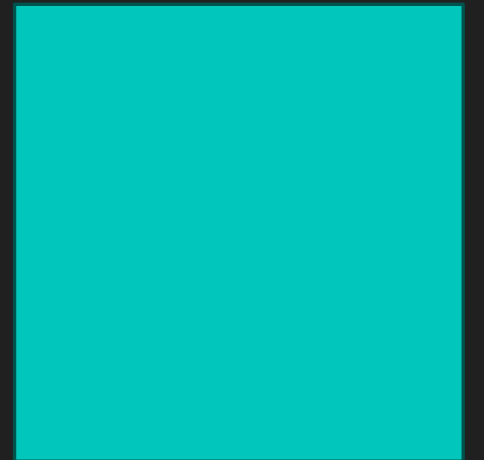
$$-4, 2$$

$$x(x-4) + 2(x-4) = 0$$

$$(x-4)(x+2) = 0$$

$$x = \underline{\underline{4, -2}}$$

$$\begin{aligned} 4 + (-2) &= 2 = \frac{-(-2)}{1} = \underline{\underline{2}} \\ 4 \cdot (-2) &= -8 = \frac{-8}{1} = \underline{\underline{-8}} \end{aligned}$$



Exercise 2.2

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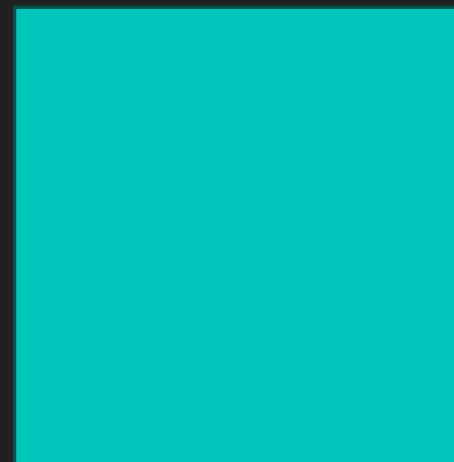
(iii) $6x^2 - 3 - 7x$

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Exercise 2.2



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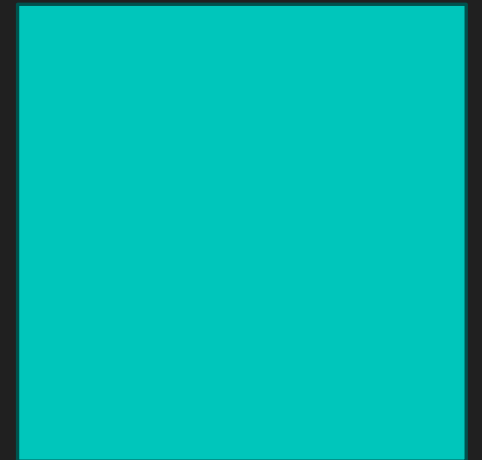
Exercise 2.2

$$ax^3 + bx^2 + cx + d = 0$$

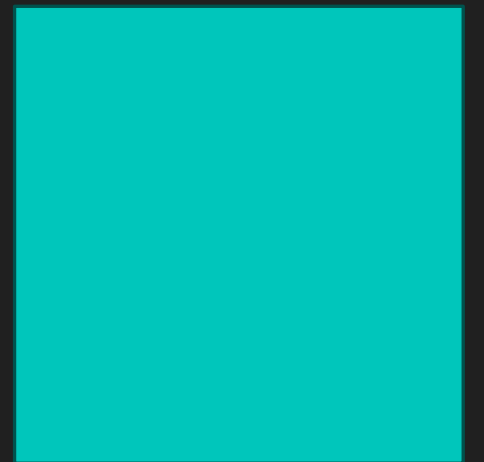
$$\alpha + \beta + \gamma = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$$

$$\alpha\beta\gamma = -d/a$$



Division of a Polynomial



Division of a Polynomial

